

Birkbeck BA Philosophy, Logic and Metaphysics 2001–2002, Term 1, Weeks 1–5

Truth

Backup Class Notes on Truth and the Liar Paradox, including a summary of ch 5, §§2–8 of R.M. Sainsbury Paradoxes (2nd Edition)

Truth and the Liar Paradox

(L1) L1 is false

Suppose L1 is true, then it's false.

Suppose it's false. L1 says it's false, and if a sentence tells it like it is, it's true, so it's true.

Summary:

If L1 is true, then it's false.

If L1 is false, then it's true.

If we assume:

If p is false, then p is not true.

If p is true, then p is not false.

We get:

If L1 is true, then it's not true.

If L1 is false, then it's not false.

But the following principle of inference holds:

(If p then not-p) entails not-p

So we can conclude:

(G) L1 is neither true nor false.

This isn't paradoxical, unless we have independent reason to suppose that L1 is either true or false.

The principle:

Every sentence is true or false.

is problematic (questioning sentences are not true or false, for instance), but it looks like something like the principle is right:

Any non-defective representation of how things are in the world must be either accurate or inaccurate, true or false.

Some sentences look seem to be semantically defective: consider

That elephant is about to charge.

said when there's no elephant about.

So, one natural response to the Liar is to accept (G), and say that L1 is neither true nor false, but we need to justify this claim. One way is to say that L1 is semantically defective in some way.

Grounding and Truth

Key proposal: Truth must be grounded in something outside the sentence.

Suppose we'd learned this about truth:

Call a sentence true iff you are willing to assert it.

What does this tell us about the following?—

“Snow is white” is true.

If we think that snow is white, it tells us that the claim is true, but only once we've gone down to the level of “snow is white”—down to ground level where the sentence makes a connection with things outside itself. Applying this idea to Liar sentences, we can try claiming that they are defective in that with them we never get to ground level—they are “ungrounded”.

The Strengthened Liar

[First] Unfortunately, it seems that (G) itself generates a paradox.

Recall, we had

(L1) L1 is false

and

(G) L1 is neither true nor false

Now, (G) entails

(not- L1) L1 is not false.

But this (that is (not- L1)) is a negation—it's just

It's not the case that (L1 is false)

But now consider the following principle

(Princ) Where “It’s not the case that p” is true, “p” is false

Now, in the case of

It’s not the case that (L1 is false)

what’s gone in place of “p” is “L1 is false”, but that’s (L1), so we can infer

L1 is false

and we have

L1 is not false and L1 is false

[Second] (G) cannot deal with

(LG) LG is either false, or neither true nor false.

Suppose LG is neither true nor false. It follows that LG is true (because “p or q” is true iff “p” is true or “q” is true). And if LG is true, it follows that it is either true or false.

Suppose LG is true, then either it’s false, or neither true nor false. Since it’s true it’s not false, so it’s neither true nor false, so it’s not true. So, if it’s true, then it’s not true. So, it’s not true.

Suppose LG is false, then it will have to be both not false and not neither true nor false. But that means it has to be not false. So if it’s false, then it’s not false. So it’s not false.

Putting the last two main conclusions together, LG is neither true nor false.

Finally, we have

LG is neither true nor false and LG is either true or false.

[Third] The Strengthened Liar

(L2) L2 is not true.

Suppose L2 is true, then it’s not true, so it’s not true.

Suppose L2 is not true, then it tells it the way it is, so it’s true. So...

If L2 is true, then it’s not true.

If L2 is not true, then it's true.

So

L2 is true iff L2 is not true

Can (G) help us here?

(G) L2 is neither true nor false

this entails

L2 is not true

but then it follows that L2 is true (by "L2 is true iff L2 is not true").

Reasoning like this can be taken to show that the "semantic defect" approach is doomed, because whatever is semantically defective is (thereby) not true.

Levels

We've been relying a lot on the following principles:

If a sentence is true, then things are as it says they are

If things are as a sentence says they are, then the sentence is true

We can put this more formally. First,

Use "s" to stand in for a sentence name

Use "p" to stand in for a sentence

Tarski says that for any acceptable language, we must accept all instances of the following form in which the sentence named by s means the same as p.

(T) s is true iff p

This seems completely obvious. Take an example

"Snow is white" is true iff snow is white

But the strengthened liar shows that (T) has contradictory instances in English:

(*) L2 is true iff L2 is not true

(“L2” is the name of a sentence which means the same as “L2 is not true”.)

“Tarski’s response is that the ordinary concept of truth... is incoherent... [and must be replaced] by a series of concepts of truth, hierarchically arranged, and each expressed in a language different from any natural language...”

The problem is meant to be that English contains a term “true” which can be applied to all (declarative) sentences of the language, even ones which contain the term itself.

Tarski proposes, in effect, that we start with a language which does not contain a truth-predicate, then we can have a language which expands on that, containing a truth-predicate which can be applied to any sentence of the starting language, but not to sentences of the expanded language, then we can have a language which expands on that, and so on...

The problem with this is that our ordinary conception of truth just does have it being applicable to all claims. A proposal slightly less radical than Tarski’s would be that a hierarchy is implied in our actual use of “is true”, but this proposal just seems wrong. For instance, consider the following case

B says: Snow is white. “Grass is green” is true.

C says: Everything B just said is true.

What level is C’s “true” meant to be at?

Self-Reference

Self-reference seems to be an important ingredient in lots of Liar puzzles. In

(L2) L2 is not true

the sentence L2 contains a reference to itself. Is self-reference the source of the trouble? Might we deal with the puzzles by showing self-reference to be a defect?

Not all Liar-type puzzles involve self-reference:

? says on Monday: (A) everything ? will say on Tuesday is true.

? says on Tuesday: (B) Nothing ? said on Monday is true.

Supposing ? says nothing more on Monday and ? says nothing more on Tuesday, this leads to paradox.

Now, it’s not that the sentences which ? and ? use are deprived of meaning. To see this note that a third person might say

(B) Nothing ? said on Monday is true.

And this would be perfectly meaningful.

Indexicality

Perhaps the root of the problem is that Liar paradoxes arise when something goes wrong with an indexical element?

First, introduce the distinction between sentences and things said by uses of sentences (statements). “I am hungry” is a meaningful sentence of English, but it may be used to make different statements. When Tom says “I am hungry” he makes a statement which is true iff he (Tom) is hungry. When Dick says “I am hungry” he makes a statement which is true iff he (Dick) is hungry.

Some uses of meaningful sentences can fail to make statements. Suppose I shout

That elephant is about to charge.

when no elephant is in view. It seems I’ve failed to make a statement. Perhaps Liar puzzles involve similar failures. Consider

(L2*) L2* does not express a true statement.

We might think that this somehow failed to express a statement. Put it this way, suppose I say

This statement is false

intending “this” to refer to the statement I’m in the process of making. Can’t we say with some rhetorical force “What statement? What is it that you’re saying is false?”.

If that’s right, then it would seem that the puzzle is defused—paradox can’t get going in this way.

Or can it? If we think that L2* is defective, we might say

L2* does not express a true statement

And it might worry us that we’ve used the sentence L2* to assert our view. That is, taking our claim seriously, we think we’ve used it to say something true. The worry would be that paradox will result. But this needn’t happen. Because we’re distinguishing sentences from statements, we can claim that the first use of “L2* does not express a true statement” is faulty, and does not express a statement, whereas

the second use is not faulty, and expresses a statement which is true.

The main problem to be faced in trying to articulate a solution of this form is pressing home the idea that Liar paradoxes involve some kind of failure involving indexicality. A satisfying solution would provide an account of the kind of failure involved. One possibility here is Russell's Vicious Circle Principle:

(VCP) No totality can contain members fully specifiable only in terms of itself.

If we consider

(Lx) (The statement) Lx is not true

we might say that the totality (of statements) involved is (just) Lx, but this seems specifiable only in terms of itself. VCP would rule that there was no such statement.

This approach looks vulnerable. It's hard to find independent motivation for VCP. And some Liar puzzles don't seem to fall to it, e.g.

No use of this very sentence expresses a true statement.

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