UNIVERSITY OF LONDON

088 0320

BA EXAMINATION

for Internal Students

This paper is also taken by Combined Studies Students

PHILOSOPHY

Optional Subject (i): Set Theory and Further Logic

Monday, 28 April 2008:10.00-13.00

Answer THREE questions, at least ONE from EACH section

SECTION A

1. (i) Show that not all instances of the following schema are true, where "$F(x)$" is schematic for a predicate:

$$\exists B \forall x [x \in B \leftrightarrow F(x)].$$

(ii) Prove that there is no universal set.

(iii) Prove that for any set A there is no set of everything not in A.

(iv) Prove that if there is a set, there is an empty set.

(v) What is it for a two-place predicate $\Psi(x, y)$ to be univalent on a set B? Prove that the predicate "$x = \{y\}$" is univalent on any non-empty set B. State the Axiom Schema of Replacement and use it to show that there is no set of all unit sets.

TURN OVER

8. (i) Prove that any infinite ordinal α is equinumerous to its successor α^+.

(ii) Define the cardinal $|A|$ of a set A. What is it for an ordinal to be a limit ordinal? Prove that every infinite cardinal is a limit ordinal.

(iii) State the theorem that legitimates definition by transfinite recursion on the ordinals. Define the aleph operator $\aleph$ by transfinite recursion on the ordinals. (You may assume that for any set B of cardinals there is an infinite cardinal not in B.)

(iv) Prove that for any ordinals α and β, if $\alpha \in \beta$ then $\aleph_\alpha < \aleph_\beta$.

SECTION B

9. (i) Is the following valid in T: $\Diamond(p \wedge q) \equiv (\Diamond p \wedge \Diamond q)$? Justify your answer.

(ii) Show that the characteristic thesis of the Brouwerian system, $p \supset \Box\Diamond P$, is not valid in S4.

(iii) (a) Provide a model to show that $(\Box p \supset \Box\Diamond\Box p)$ is not valid in T.

(b) Provide a model that shows that $(\Box\Diamond\Box p \supset \Box p)$ is not valid in S4.

(iv) Provide an S5 model in which $\Box\exists x Px \supset \exists x \Box Px$ is false at some world.

(v) Explain why, if domains are allowed to vary freely, the converse of the Barcan Formula is not valid even in T.

10. Let three frames $(G_1, R_1), (G_2, R_2).(G_3, R_3)$, be defined by

$G_1 = \{a\},\ R_1 = \{\langle a, a\rangle\}$
$G_2 = \{a, b\},\ R_2 = \{\langle a, b\rangle, \langle b, a\rangle, \langle b, b\rangle\}$
$G_3 = \{a, b\},\ R_3 = \{\langle a, a\rangle, \langle a, b\rangle, \langle b, b\rangle\}$

($\langle x, y\rangle$ in R_i means xR_iy).

For each of the three frames determine which of the following formulas are valid on the frame.

(a) $P \supset \Diamond P$

(b) $\Diamond\Diamond P \supset \Diamond P$

(c) $\Diamond\Box P \supset P$

(d) $\Box P \supset \Diamond P$

(e) $\Box\Box P \supset \Box P$

Moreover, if one of the formulas ϕ is not valid on frame (G_i, R_i), give a world x in G_i and a forcing relation $\Vdash$ between G_i and $\{P\}$ such that $x \Vdash \neg\phi$.

11. Let (G, R, D) be the following frame: $G = \{0, 1, 2, 3 \ldots\}$ is the set of natural numbers. nRm holds if n is a smaller number than m, and D is a function assigning to every element of G a non-empty set. Let $(G, R, D, \Vdash)$ be a model over (G, R, D) and let the function v assign to every variable an element of the domain, $\bigcup\{D(n) \mid n \in G\}$, of the model.

(i) Argue that $(\Diamond P \wedge \Diamond Q) \supset \Diamond((\Diamond P \wedge Q) \vee (P \wedge \Diamond Q) \vee (P \wedge Q))$ is valid on the frame (G, R).

(ii) Argue that, if $D(0)$ is a *finite* domain and all Barcan formulas are valid on this model, then $\Diamond\Box\phi$ is valid, for any ϕ that is a converse Barcan formula.

(iii) Suppose $(G, R) \Vdash_v \phi(y)$, where $\phi(y)$ is one of the formulas

(a) $\Box\Diamond(\exists x)(x = y)$,

(b) $\Diamond\Box(\exists x)(x = y)$,

(c) $(\exists x)(x = y) \supset (\exists x)\Box(x = y)$.

For each of the formulas, explain what properties the domain element $v(y)$ satisfies.

Question continues on next page

TURN OVER

(iv) Let D assign the same non-empty set to every natural number and let $\phi(y)$ be a property of natural numbers, such that $(\forall y)(\Box\Diamond\phi(y) \supset (\phi(y) \wedge \Box\phi(y))$ is valid on the model. What does this say about the property $\phi(y)$? (hint: as any implication $(P \supset Q)$ is logically equivalent to $(\neg P \vee Q)$, there are two cases to check).

12. Use the propositional tableau proof systems to prove the following three formulas

(a) $\Box(p \supset \Diamond p)$ in the **T** system,

(b) $\Diamond(\Diamond p \vee \Diamond q) \supset (\Diamond p \vee \Diamond q)$ in the system **S4**,

(c) $\neg((\Box(\Diamond p \supset p) \wedge \Diamond(\Diamond\neg p \wedge p))$ in the system **S5**.

Use the constant domain tableau system to determine whether (d) is valid on all **K** models with constant domain

(d) $(\Diamond(\forall x)A(x) \wedge \Box(\exists x)B(x)) \supset (\exists x)\Diamond(A(x) \wedge B(x))$.

Use the variable domain tableau system to determine whether (e) is valid on all **K** models with varying domain

(e) $(\Diamond(\forall x)A(x) \wedge \Box(\exists x)(B(x)) \supset \Diamond(\exists x)(A(x) \wedge B(x))$.

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