UNIVERSITY OF LONDON

BA EXAMINATION

for Internal Students

This paper is also taken by Combined Studies Students

PHILOSOPHY

Optional Subject (i): Set Theory and Further Logic

Answer THREE questions, at least ONE from EACH section

SECTION A

SECTION B

1. Let three frames $(G_1, R_1), (G_2, R_2).(G_3, R_3)$, be defined by

   $G_1 = \{a, b, c\},\ R_1 = \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$
   $G_2 = \{a, b, c\},\ R_2 = \{\langle a, b\rangle, \langle b, a\rangle, \langle b, c\rangle, \langle c, b\rangle, \langle a, c\rangle, \langle c, a\rangle\}$
   $G_3 = \{a, b, c\},\ R_3 = \{\langle a, b\rangle, \langle c, b\rangle, \langle a, c\rangle, \langle c, a\rangle, \langle a, a\rangle\}$

   ($\langle x, y\rangle$ in R_i means xR_iy). For each of the three fames determine which of the following formulas is/are valid on the frame.

   (a) $\Box P \supset P$

   (b) $\Box P \supset \Box\Box P$

   (c) $P \supset \Box\Diamond P$

   (d) $\Box P \supset \Diamond P$

   Moreover, if one of the formulas ϕ is not valid on frame (G_i, R_i), give a world x in G_i and a forcing relation $\vdash$ between G_i and $\{P\}$ such that $x \vdash \neg\phi$.

TURN OVER

2. Use the propositional tableau proof systems to prove the following three formulas

   (a) $\neg\Box(\Diamond\neg\phi \wedge \phi)$ in the **T** system

   (b) $\Diamond\Box(\Diamond\Box\phi \vee \Diamond\Box\psi) \supset (\Diamond\Box\phi \vee \Diamond\Box\psi)$ in the system **S**4,

   (c) $\Diamond\neg\phi \supset \Box(\Box\phi \supset \psi)$ in the system **S**5,

   Use the constant domain tableau system to determine whether (d) is valid on all **K** models with constant domain

   (d) $(\Diamond(\forall x)A(x) \wedge \Box(\exists x)B(x)) \supset (\exists x)\Diamond(A(x) \wedge B(x))$

   Use the variable domain tableau system to determine whether (e) is valid on all **K** models with varying domain

   (e) $(\Diamond(\forall x)A(x) \wedge \Box(\exists x)B(x)) \supset \Diamond(\exists x)(A(x) \wedge B(x))$.

3. a. Using the varying domain rules and the equality rules give a tableau proof of $(\exists x)\Diamond P(x) \wedge \Box(\forall x)\neg P(x) \supset (\exists x)\Diamond(\forall y)(x \neq y)$

   b. Use the Tableau method to construct a normal, varying domain **K** counter model for the sentence $(\forall x)\Box(\exists y)(x = y) \supset ((\forall x)\Box P(x) \supset \Box(\forall x)P(x)))$

4. a. Show that there is a constant domain model allowing predicate abstraction in which $\Diamond\langle\lambda x.P(x)\rangle(c) \wedge \neg\langle\lambda x.\Diamond P(x)\rangle(c)$ can be satisfied.

   b. Show that $(\forall y)\Diamond\langle\lambda x.P(x)\rangle(y) \wedge \neg\langle\lambda x.\Diamond P(x)\rangle(c)$ cannot be satisfied in such a model.

5. Considering $\Box, \Diamond$ as a temporal operators, where $\Box P$ is interpreted as "P is and will always be the case" give a formalization using predicate abstraction of the sentence "Someday the prime minister won't be the prime minister (anymore)" and discuss how a logical contradiction is avoided.

END OF PAPER