What is the most effective way to teach maths to online undergraduate students?

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Introduction

Students on the University of London's online BSc Computer Science programme are required to pass three Level 4 maths modules. The modules are delivered on the Coursera platform. As part of a review of student feedback CODE was asked to review good practice in teaching mathematics online with a view to providing good practice recommendations to the leaning design team.

The Coursera modules share a common structure that provides a repeated cycle of short video / quiz / reading. Videos are transcribed for accessibility. After watching a video, students are asked to engage with further reading, work through some practice examples and check their understanding through a quiz. When a student has made a mistake, automated feedback prompts them to revisit a particular piece of text.

The project team considered the following questions:

- Does teaching maths online needs a different learning design approach to teaching other content, or are the learning design principles the same?
- What is the most effective way to teach maths on online undergraduate courses?
- Are there any specific or specialised elements to maths teaching that need to be taken into consideration?

Mathematics and computer science are conceptually quite close, but computer science professionals use mathematics relatively rarely in their practice. As a result, teaching mathematics for computer science poses pedagogical and motivational challenges in addition to those faced in mainstream online mathematics teaching. The most recent overview of these issues that we could find is a paper by Baldwin et al (2013).

The project team reviewed the Coursera materials and analysed student feedback over four presentations of each module. One of the Code Fellows had four decades of experience of teaching mathematics online and the Student Fellow was a student on the BSc Computer Science. Each contributed a reflective account of their experience. The team also undertook a literature review focused on 'mathematics teaching' in general, 'mathematics teaching online' and 'mathematics for computer science'.

In the remainder of this report, we summarise the team's findings and provide suggestions for good practice.

Literature review

The academic literature on mathematics teaching in higher education is relatively sparse, more so when the field is restricted to online and distance education. There was, however, a significant spike in contributions in the aftermath of the Covid 19 lockdowns when most in person, campus-based mathematics teaching was forced to move online.

A 2019 paper by Abdulwahed et al provides a useful overview of university mathematics teaching in general and notes (p49) that it

> *'… has long embraced traditional methods: non-interactive ways of teaching mathematics '.*

They find some evidence of a shift towards constructivist pedagogy and summarise this as an approach where (p50):

> *' … learning is a student-centred process, students' autonomy should be fostered; learning should be contextualised and associated with authentic real-world environments and examples; social interaction and discourse form an important part of learning; the taught elements should be made relevant to the learner; the taught elements should be linked with the learners' previous knowledge; it is important to facilitate continuous formative assessment mechanisms, self-esteem and motivation; teachers should act as orchestra synchronisers rather than speech givers; and teachers should consider multiple representations of their teachings.'*

As online teaching became more common in the first decade of the twenty first century researchers and practitioners picked up on the pedagogical challenges for teaching mathematics. Engelbrecht and Harding (2005, p256) observed that in some instances web-based teaching could be quite narrowly instructivist.

> *'There exist web courses that are still based on lectures that are videotaped and students watch the lectures when and where they can, reviewing the same part as many times as they want. This teaching style is not much different from the instructivist style of teaching – in fact, one could argue that web-based courses such as these could provoke a return to a backward pedagogy, with learners' participation reduced to reading and individual work on exercises.'*

In contrast in the context of online and distance education Mason et al, (2005, px) stress the importance of active learning

> *'Really is it enough just read through materials. Learning is a process of maturation like a fine wine or a good cheese: it takes place over time rather than just at the time of study. Full engagement comes about by actively 'doing' things such things as: jotting down ideas; doing tasks and constructing your own examples; trying to make connections; getting involved in detail; standing back to get the big picture …*

At the height of the Covid 19 pandemic most in person teaching of mathematics in higher education moved online. Cassibba et al (2021) surveyed lecturers teaching Maths online and analysed the differences between fully online and face to face. Their findings emphasise the importance of writing so that students can observe process. They also comment on the value of using software to create multiple examples and to help student visualise topics. Ní Fhloinn and Fitzmaurice (2021) surveyed lecturers from 29 countries. Around 19 per cent of the sample observed that not being able to write mathematics live as they spoke was a big disadvantage of the new online environment.

> *'Writing mathematics is a core part of how I explain mathematical arguments in my teaching, I found it* very *constraining not to be able to do this live' (p7).*

In addition, they noted that while slides were common in other disciplines they were inadequate when teaching maths. They also highlighted that there is a

> *'… need for students to see as well as hear—the visual nature of formatted mathematics'(p9)*.

The need for adequate preparation is stressed by Manacero and Marranghello (1999), not only because of the hierarchical nature of mathematical concepts, but also because some students will approach the subject with anxiety

> *'Engagement with mathematics may also be accompanied by apprehension and fear, particularly if previous experiences of mathematics have been negative, or if learners lack mathematics experience or are underprepared, lack confidence, feel anxious about mathematics, or have protected themselves by avoiding the subject …'*

They also stress the importance of contextualising maths and the use of storytelling to relate maths to computer science. This concern is echoed in Baldwin et al's (2013) critical review of the teaching of mathematics on computer science degrees. Baldwin et al also emphasise the challenge of teaching mathematics for computer science when the two disciplines intersect so strongly at the conceptual level:

> *'Yet, paradoxically, many computer science and software engineering graduates function quite well as professionals without consciously applying mathematics to their work.'(p79)*

They suggest that:

> *'The time has come for all of us, as computer science and software engineering educators, to reform the role of mathematics in our curricula.' (p80)*

Discussion

Students of mathematics need to develop technical expertise and conceptual understanding. Our reading of the literature and reflection on our own practice suggests that providing students with opportunities to observe the **process** of maths being done and seeing and hearing how tutors and peers tackle maths problems are critical to the development of conceptual understanding. This modelling of practice may happen more organically in person but needs to be purposefully created in distance learning settings. These observations resonate with the feedback from students that we were able to review, which was dominated by comments on the module videos. Typically, students wanted more explanation of process and more modelling of good practice. Some students also noted that they had made use of sources outside of the module materials, particularly from the Khan Academy and YouTube. It's notable that these resources that students value are often more explanatory in style.

Technical competence also requires opportunities to practice and to learn from mistakes. In their feedback students requested opportunities to practice more. In a reflective account of her experience as a student on the Computer Science degree the Student Fellow remarked that:

> *There are too few practice exercises. The more we practice, the better our understanding will be since Math requires critical thinking and problem-solving abilities by applying those concepts.*

To internalise technique, develop fluency and develop conceptual understanding it's necessary to work actively with multiple examples.

The affective dimension of study is also important. It's not uncommon that students embark on further mathematics study with negative prior experience. And in the context of Computer Science, they may also need to be convinced of the need to study maths at all. There is scope for greater use of contextualisation in the Coursera mathematics models with the provision of explicit links between topics, skills and their applications in

Computer Science. In an online study environment these links need to be clear and explicit since there are fewer opportunities to pick them up informally.

Students on the Computer Science degree are required to have studied successfully up to English AS level in a mathematical subject or GCSE grade B (or equivalent). However, while formally students begin their study with equivalent prior experience, in practice in any multinational student body studying online there are likely to be variations in experience and use of notation. While this was not mentioned in student feedback, we would suggest that a thorough understanding of notation is important to successful learning.

Recommendations

1. **Video material should concentrate on process.** It's important for students to see the evolution of solutions and the rationale for choosing a particular approach. Simply seeing the finished product – often neatly typeset in PowerPoint – jumps over the necessarily messy and untidy stage of problem solving and encourages passive reading rather than active engagement. This is particularly important when students are working asynchronously and don't have the opportunity to question the lecturer, as may be the case in face-to-face settings. **Modelling process is more likely to support the development of conceptual understanding**, and we would argue that this is particularly important for computer science students who may have relatively few opportunities to practice their maths skills post-graduation but need to retain a grasp of concepts.
2. There are challenges in implementing constructivist pedagogy in an online environment – and in the context of mathematics teaching it's important to recognise that deep learning requires the development of a hierarchy of concepts, at the same time as acquiring fluency in the language of the subject and facility with the manipulation of mathematical objects. So along with an emphasis on process there are **two further key considerations** for learning design. **Firstly, ensuring that students are familiar with methods and notation**. Students should have comparable levels of prior experience but given that they come from many different education systems there will inevitably be gaps. These can be overcome by being careful to avoid assumptions in the course materials and hyperlinking methods and notation to a glossary. **Secondly, providing multiple opportunities for practice and making sure that feedback on errors goes beyond simply a reference back to the course material**. There are opportunities for automating the provision of examples and feedback.
3. **Students need to be confident with notation.** Hence a key issue for learning design is to ensure that notation is explained with examples (perhaps through a linked glossary).
4. **Learning design should also pay attention to the affective domain of student experience**. Some students may have had a less than positive prior experience of studying mathematics and many computer science students may question why they need to spend so much time on mathematics. **Short case studies, historical examples and anecdotes can be used to motivate learners**. **Providing software that enables students to rapidly**

visualise and experiment with functions, graphs and algorithms can also be motivational and support the development of conceptual understanding.

The need for a new vision for mathematics education in computer science?

Any modifications to online Computer Science maths teaching would do well to heed Baldwin et al's (2013) plea for reform of mathematics teaching. In the light of recent developments in AI this call has an even greater relevance. There are a hugely expanded range of software tools available to maths students. For example, Wolfram Alpha, a powerful symbolic mathematics programme, has been around since 2009 and available for students to generate solutions to most mathematics questions. Now, however, it can also provide all the steps to a solution as well as the solution itself. Other AI platforms also have this kind of capability. What would the curriculum look like if it was designed so that students could use these tools to explore and visualise concepts and check and compare working. Which old skills need to be retained and what new skills are necessary, and perhaps most importantly how should the reformed curriculum be assessed. While these considerations are outside the scope of the project, they are very relevant to the future development of the mathematics stream within the computer science degree.

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